Developments in bilinear systems modelling and control with industrial applications

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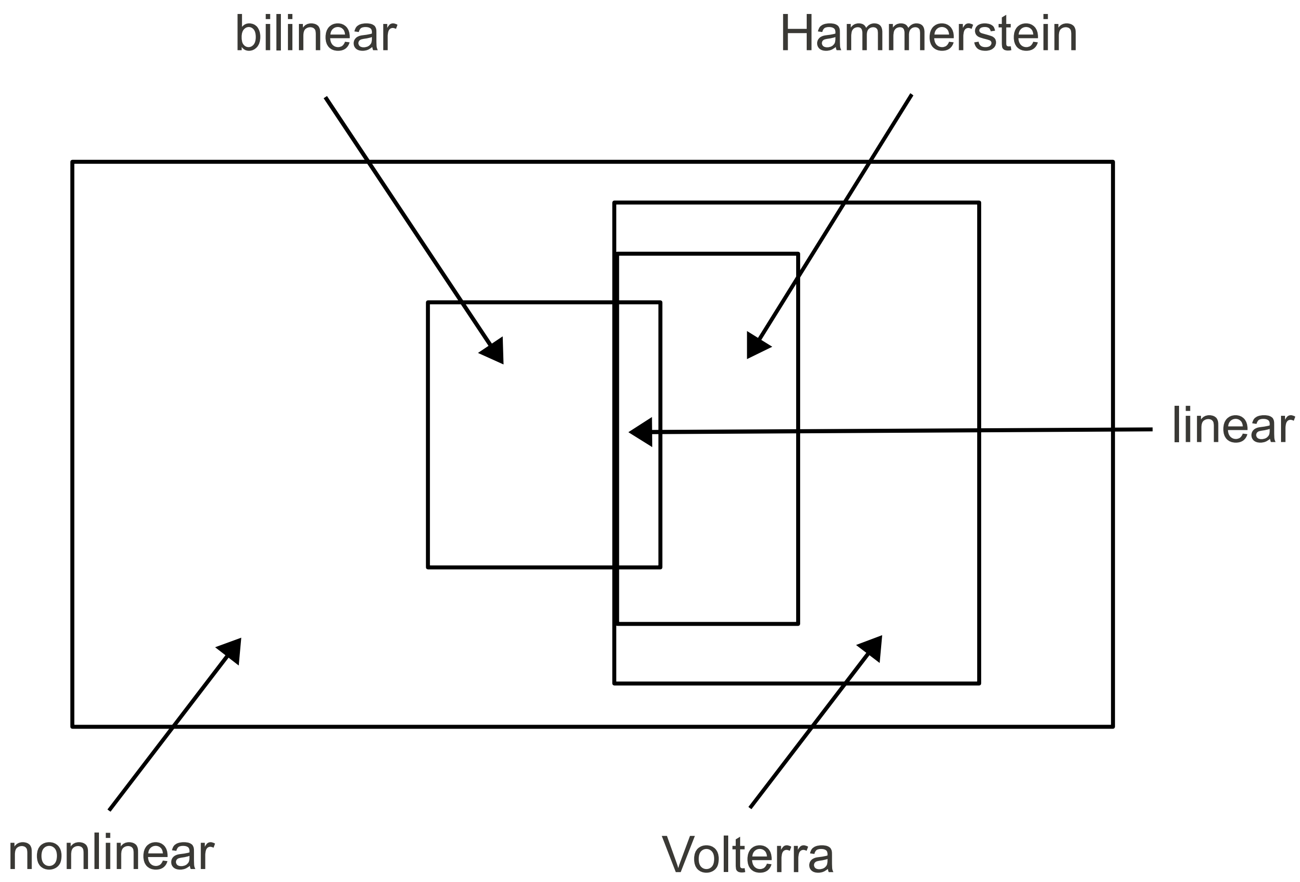
Outline of presentation

- Brief historical-technical review of control for industrial systems
- Classification of bilinear models
- Identification of first and second order bilinear model structures
- On-line parameter estimation of bilinear models - use of cautious least squares
- Application of bilinear approach to industrial systems
  - ◇ Industrial high temperature furnace (IHTF)
  - ◇ Heating ventilation & air conditioning (HVAC)
- Development of bilinear errors-in-variables (EIV) identification and filtering
- Concluding remarks & further work

Brief historical-technical review of control for industrial systems

- 1940s Three term proportional integral derivative (PID) control
- 1971-3 Optimal d-step ahead predictive schemes MV, GMV & incremental forms
- 1978-81 Sub-optimal pole-placement controllers - polynomial & state-space forms
- 1987 Long range predictive control schemes GPC
- 1987 Proportional integral plus (PIP) in Non-Minimum State-Space (NMSS)
- 1999 Bilinear PID/PIP
- 2001 Bilinear PID implemented on IHTF
- 2008 Bilinear PID implemented on HVAC
- 2009 Bilinear EIV identification

Bilinear models - classification



Bilinear models - subset of state dependent parameter models

- General form of bilinear model

$$y(t) = \sum_{i=1}^{n_a} a_i y(t-i) + \sum_{i=0}^{n_b} b_i u(t-d-i) + \sum_{j=1}^{n_a}\sum_{i=1}^{n_b} \eta_{i,j} u(t-d-i+1) y(t-d-j)$$

- General form of state dependent parameter (SDP) model

$$y(t) = \sum_{i=1}^{n_a} a_i\{\chi(t)\} y(t-i) + \sum_{i=0}^{n_b} b_i\{\chi(t)\} u(t-d-i)$$

$\chi(t)$ - non-minimal state variable vector, $a_i\{\chi(t)\}$ and $b_i\{\chi(t)\}$ - state dependent parameters

- Applications:

Industrial furnace: $y(t)$-local furnace temperature $[^oC]$, $u(t)$-gas valve position [%]

$$y(t) = -a_1 y(t-1) + b_1\{\chi(t)\} u(t-d) \qquad \text{where} \qquad b_1\{\chi(t)\} = b_1 + \eta_1 y(t-1)$$

Heating ventilation and air conditioning (HVAC) plant: $y(t)$-dew-point temperature $[^oC]$, $u_1(t)$-gas valve position [%], $u_2(t)$-outside relative humidity [%]

$$y(t) = -a_1 y(t-1) + b_1\{\chi(t)\} u_1(t-d) + o \quad \text{where} \quad b_1\{\chi(t)\} = b_1 + \eta_1 u_2(t-d) + \eta_2 y(t-1)$$

Identification of first order bilinear model structures

- Steady-state characteristics - steady-state output (Y_{SS}) and steady-state gain (SSG) given by:

$$Y_{SS} = \frac{b_0 U_{SS}}{a_0 - \eta_0 U_{SS}} \qquad \text{and} \qquad SSG_{SS} = \frac{b_0}{a_0 - \eta_0 U_{SS}}$$

  where U_{SS} - steady-state input

- First order bilinear system

$$G(s) = \frac{Y(s)}{U(s)} = \frac{b_0}{s + a_0 - \eta_0 U_{SS}} \qquad \text{with} \qquad U(s) = \frac{U_{SS}}{s}$$

- Corresponding time response to step input

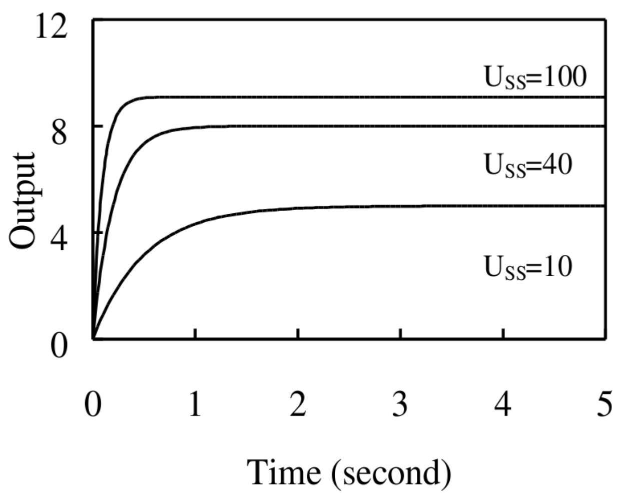
$$y(t) = \frac{b_0 U_{SS}}{a_0 - \eta_0 U_{SS}} \left(1 - e^{-t/\tau}\right) \qquad \text{with} \qquad \tau = \frac{1}{a_0 - \eta_0 U_{SS}}$$

- Simulated system:

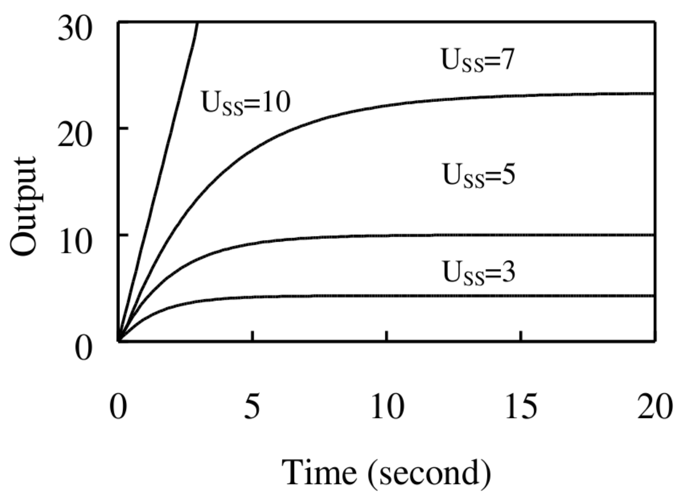
$$a_0 = 1 \qquad b_0 = 1 \qquad \eta_0 \pm 0.1$$

Identification of first order bilinear model structures

Negative bilinear term



Positive bilinear term



- Identification procedure:
  - ◇ From both steady-state output & time constant form two different step response tests
  - ◇ Bilinear model parameters calculated from:

$$\eta_0 = \frac{Y_{SS2}U_{SS1} - Y_{SS1}U_{SS2}}{\tau_1 Y_{SS2}U_{SS1}(U_{SS2} - U_{SS1})} \qquad a_0 = \frac{U_{SS2}(Y_{SS2} - Y_{SS1})}{\tau_1 Y_{SS2}(U_{SS2} - U_{SS1})} \qquad b_0 = \frac{Y_{SS1}}{\tau_1 U_{SS1}}$$

  - ◇ Note that if steady-state gains (or equivalently the time constants) equal then $\eta_0 = 0$ and system linear

Identification of second order bilinear model structures

- Consider second order bilinear system

$$G(s) = \frac{Y(s)}{U(s)} = \frac{b_0}{s^2 + (a_1 - \eta_1 U_{SS})s + (a_0 - \eta_0 U_{SS})} \qquad \text{with} \qquad U(s) = \frac{U_{SS}}{s}$$

- Corresponding time response to step input

$$y(t) = \left(1 + A_1 e^{p_1 t} + A_2 e^{p_2 t}\right) \frac{b_0 U_{SS}}{a_0 - \eta_0 U_{SS}}$$

where:

◇ input dependent constants A_1 and A_2:

$$A_{1,2} = \tfrac{1}{2} \pm \frac{\xi(U_{SS})}{2\sqrt{\xi(U_{SS})^2 - 1}}$$

◇ poles p_1 and p_2:

$$p_{1,2} = -\xi(U_{SS})\omega_0(U_{SS}) \pm \omega_0(U_{SS})\sqrt{\xi(U_{SS})^2 - 1}$$

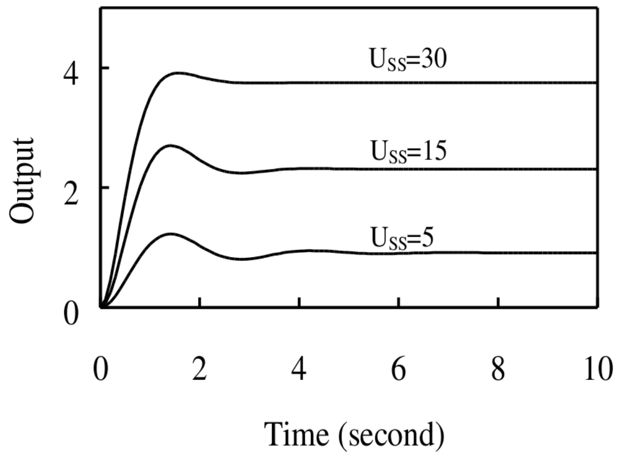
◇ undamped natural frequency ω_0 and damping ratio ξ:

$$\omega_0(U_{SS}) = \sqrt{a_0 - \eta_0 U_{SS}} \qquad \xi(U_{SS}) = \frac{a_1 - \eta_1 U_{SS}}{2\sqrt{a_0 - \eta_0 U_{SS}}}$$

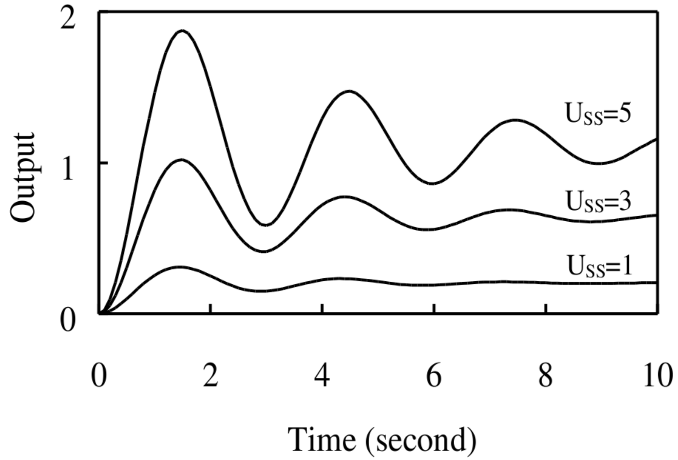
Identification of second order bilinear model structures

Simulated system: $a_0 = 5,\ a_1 = 1,\ b_0 = 1,\ \eta_0 = \eta_1 = \pm 1$

Negative bilinear terms



Positive bilinear terms

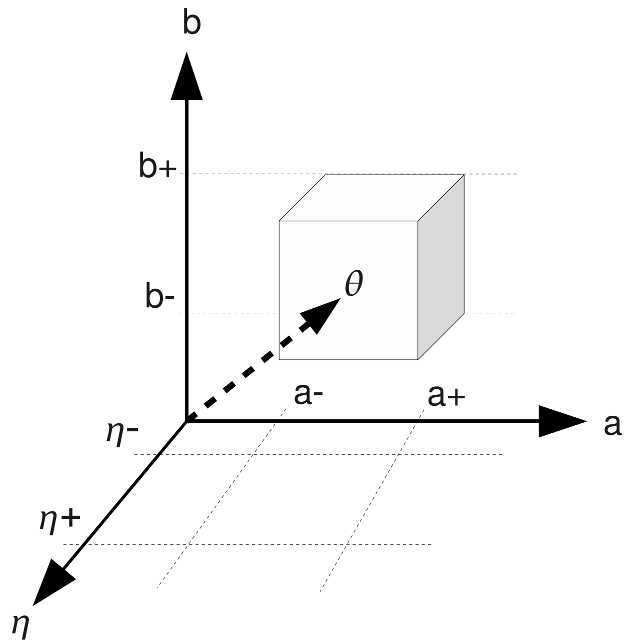


- Identification procedure:
  - Assuming that ξ and ω known for two different step inputs 1 & 2 bilinear model parameters calculated from:

$$a_0 = \frac{\omega_{n2}^2 U_{SS1} - \omega_{n1}^2 U_{SS2}}{U_{SS1} - U_{SS2}} \qquad a_1 = \frac{2\left(\xi_2 \omega_{n2} U_{SS1} - \xi_1 \omega_{n1} U_{SS2}\right)}{U_{SS1} - U_{SS2}} \qquad b_0 = a_0 \frac{Y_{SS1}}{U_{SS1}} - \eta_0 Y_{SS1}$$

$$\eta_0 = \frac{\omega_{n2}^2 - \omega_{n1}^2}{U_{SS1} - U_{SS2}} \qquad \eta_1 = \frac{2\left(\xi_2 \omega_{n2} - \xi_1 \omega_{n1}\right)}{U_{SS1} - U_{SS2}}$$

Parameter estimation of bilinear models - use of cautious least squares



- Cautious least squares - based on composite cost function derived from least squares

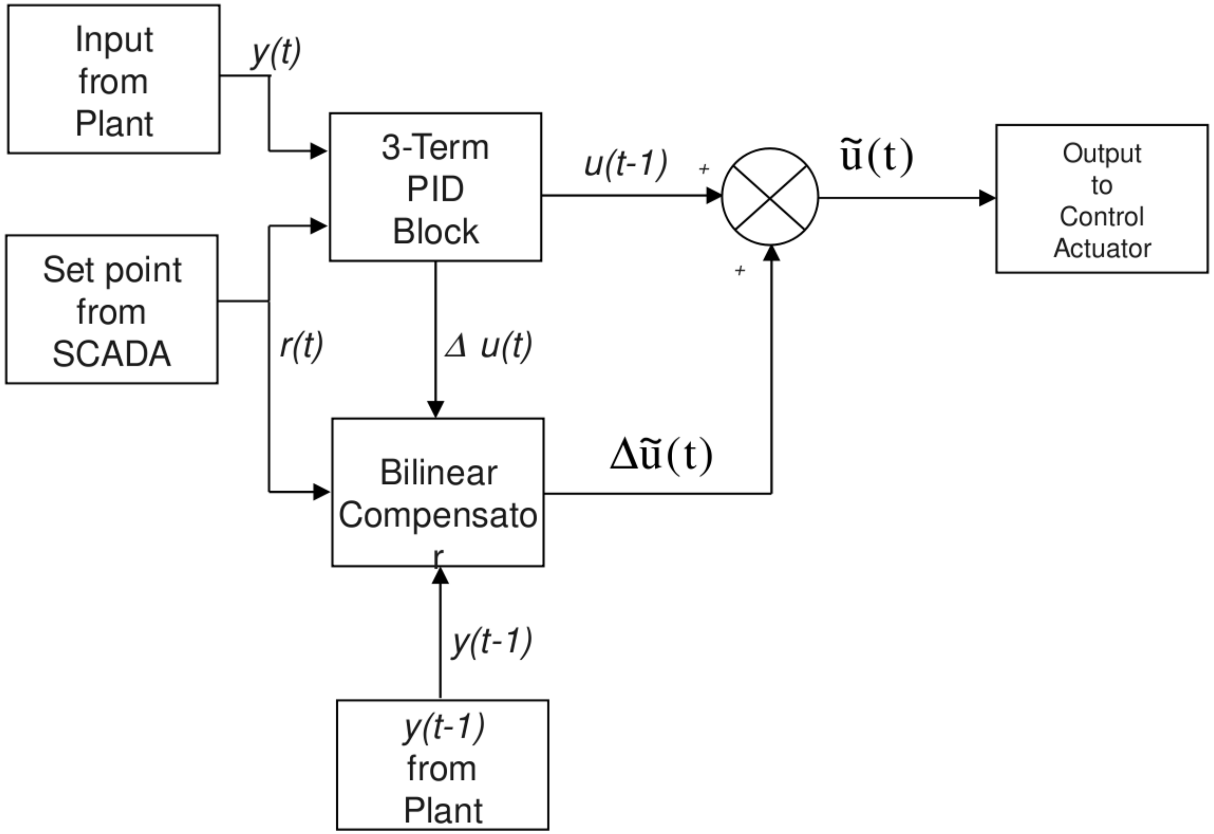
$$J(\hat{\theta}) = J_1 + J_2 = \left(y - X\hat{\theta}\right)^T \Lambda \left(y - X\hat{\theta}\right) + \left(\theta_s - \hat{\theta}\right)^T \Psi \left(\theta_s - \hat{\theta}\right)$$

where:

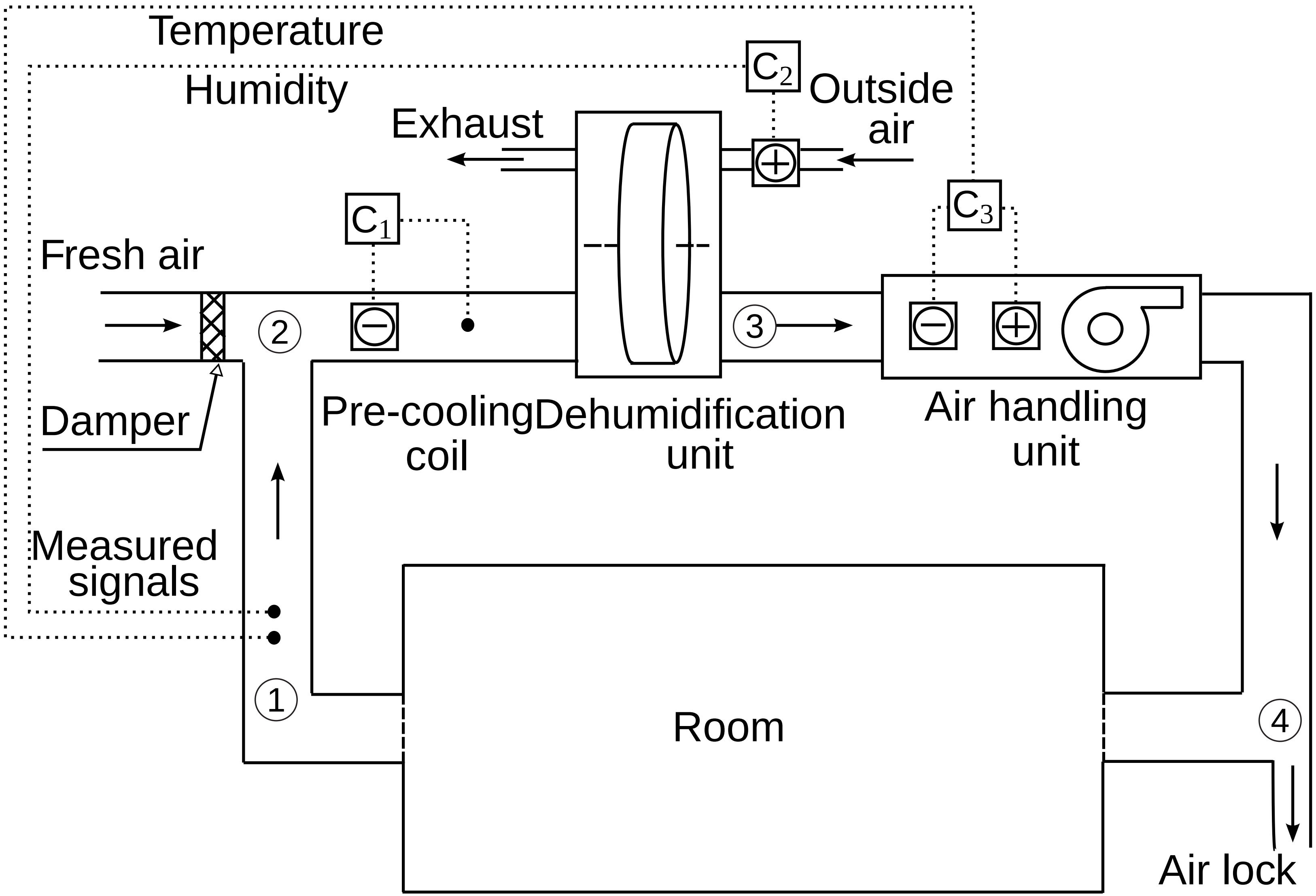
$\hat{\theta}$	-	estimate of θ	y	-	measured output
X	-	observation matrix	θ_s	-	safe set for θ representing *a priori* knowledge
Λ, Ψ	-	weighting matrices			

The Flight Electronics Heater - Fan System (laboratory bilinear system)

Pragmatic Bilinear PID as natural realisation of SDP-PIP

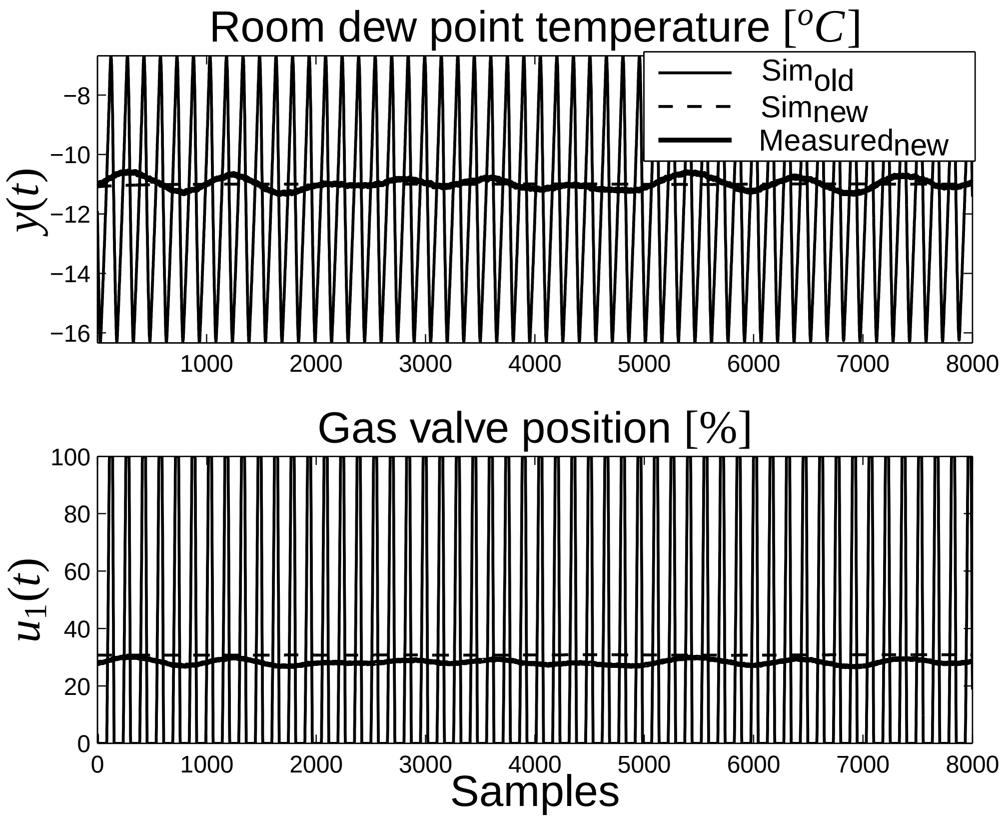


HVAC plant - block diagram



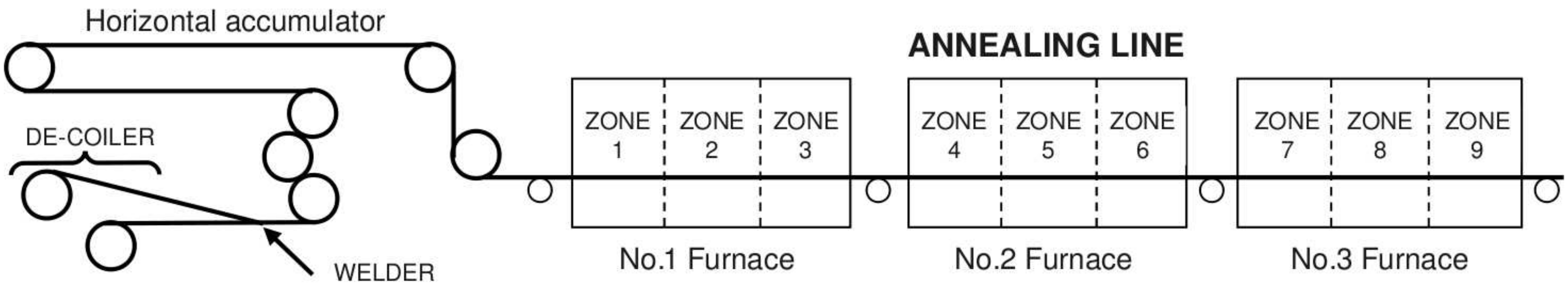
HVAC plant - air handling unit & dehumidification unit

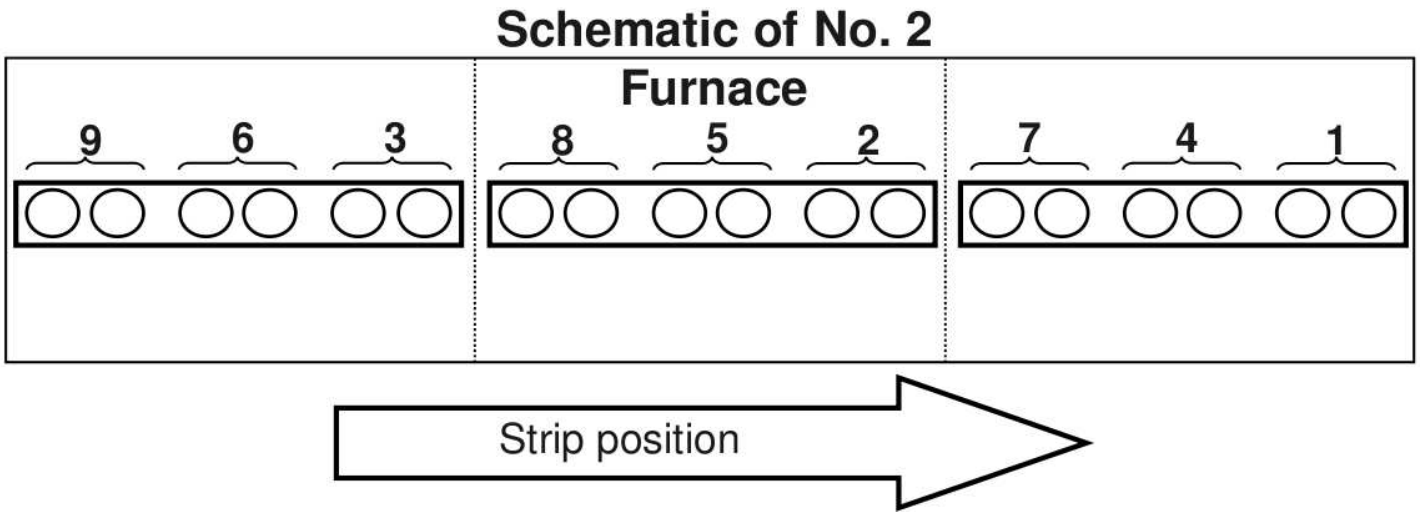
HVAC plant - performance after optimising controller gains



- Simulation of system using old PI gains - $\mathrm{Sim}_{\mathrm{old}}$, optimised gains - $\mathrm{Sim}_{\mathrm{new}}$
- Actual output using optimised gains - $\mathrm{Measured}_{\mathrm{new}}$
- Actual savings in excess of 25% so far

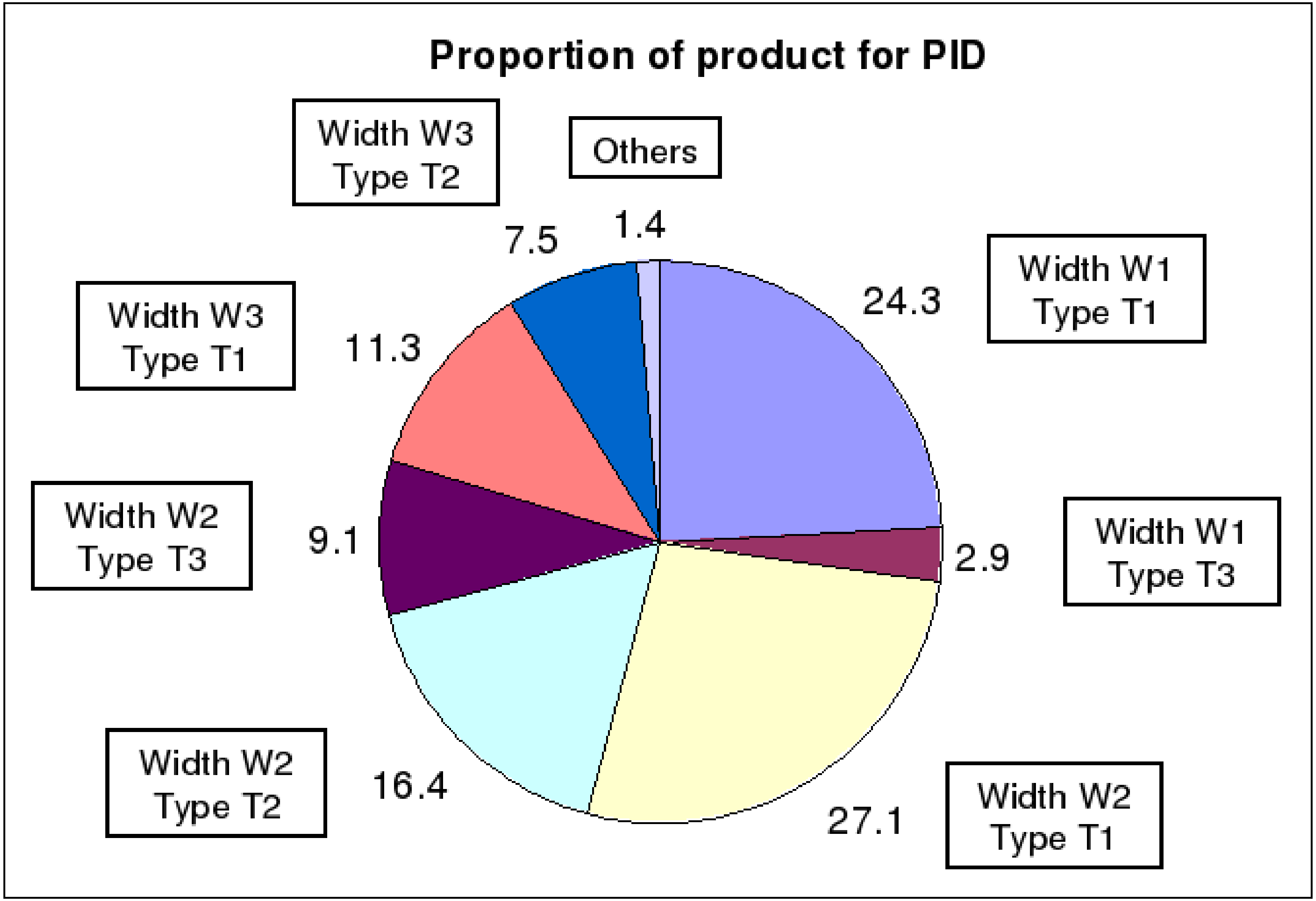
IHTF - block diagram



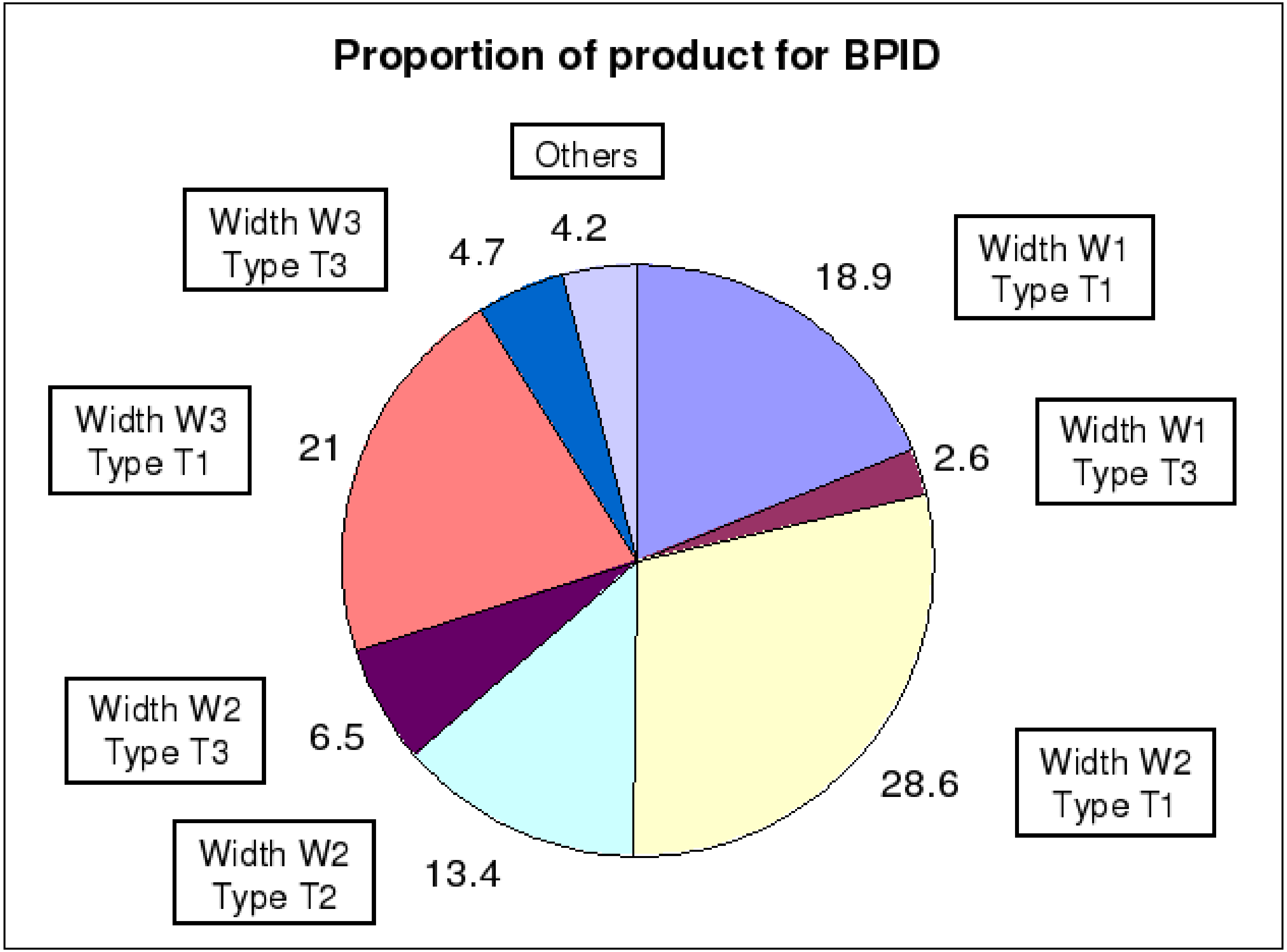


IHTF - actual plant

IHTF - longer term trials



IHTF - longer term trials

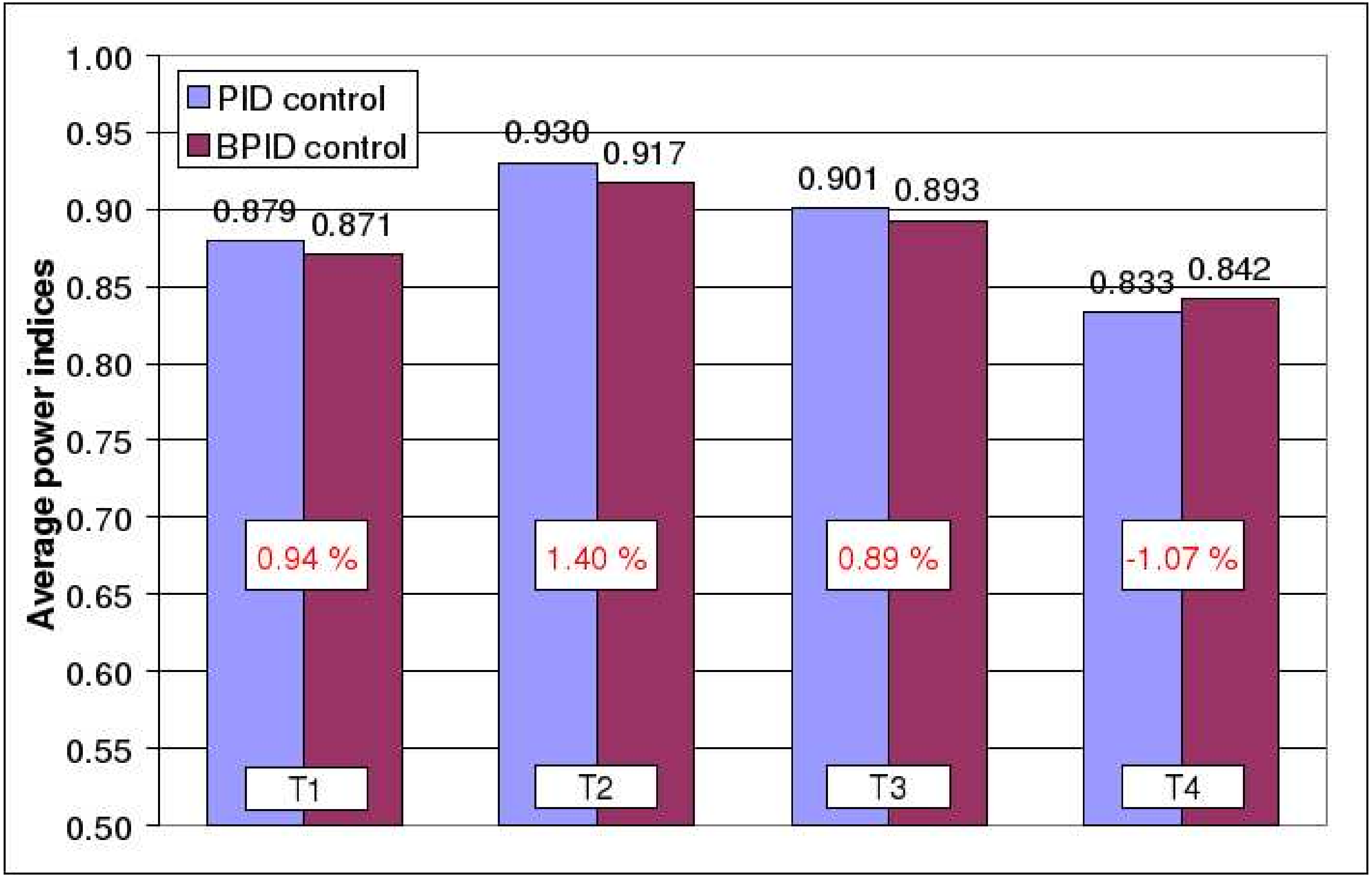


IHTF - longer term trials

Average power indices for different widths of steel

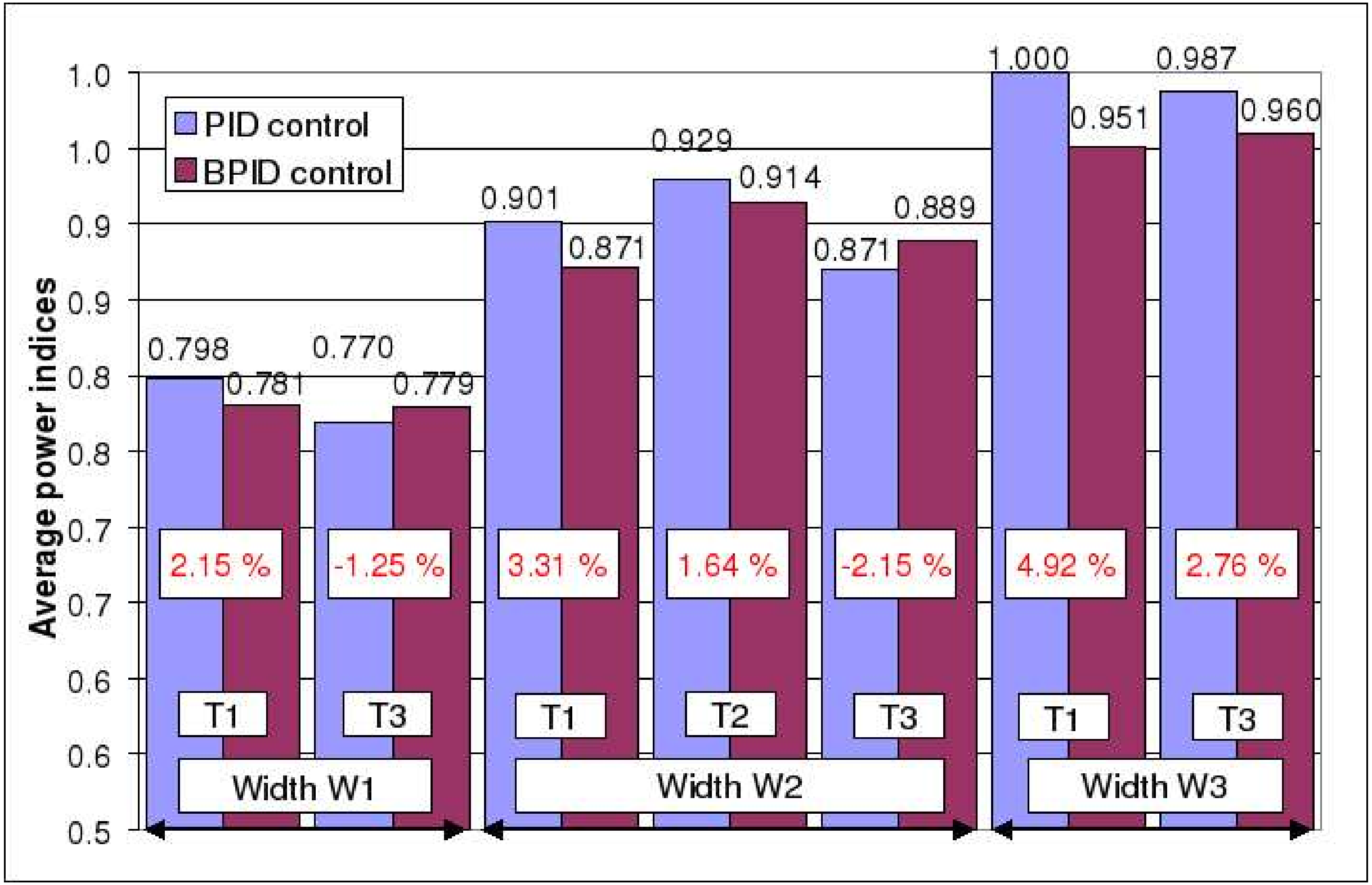
IHTF - longer term trials

Average power indices for different types of steel



IHTF - longer term trials

Average power indices for PID & BPID control strategies

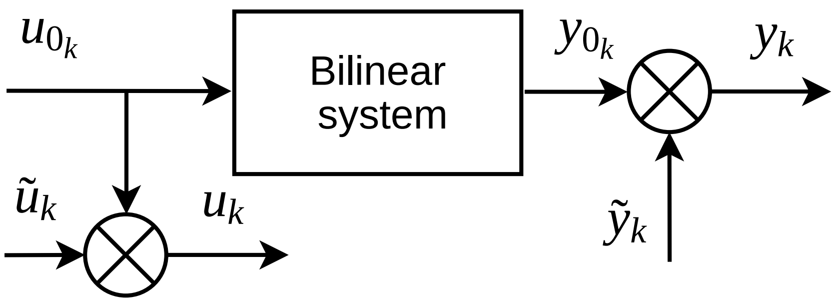


On relation between BGPC and SDP-PIP

- Model based predictive schemes
- Effective automatic gain scheduling via non-linear controller model structure
- Use made of measured input/output signals
- Analysis feasible using tools developed for linear control theory
- Structure of SDP-PIP more flexible than BGPC
- In fact BGPC special case of SDP-PIP

Errors-in-variables & bilinear models

- Errors-in-variables setup:



u_{0_k}, y_{0_k}: noise free input/output

$\tilde{u}_k, \tilde{y}_k$: input/output measurement noise

u_k, y_k: measured system input/output

$$u_k = u_{0_k} + \tilde{u}_k$$
$$y_k = y_{0_k} + \tilde{y}_k$$

- Symmetric treatment of measurements
- Potential benefits for fault detection & diagnosis
- Additional insight via estimation of noise properties
- More precise estimation of variables having physical meaning

Concluding remarks & further work

- Review of historical-technical developments in industrial context
- Recognition of real-world systems prompts premise for nonlinear approaches
- Bilinear approach builds on well established mathematical theory and development for linear systems
- Bilinear approach represents first step towards meeting demands of real-world systems whilst retaining linear systems as special subclass
- State-dependent parameter systems represent wider range of real-world systems with bilinear systems as special subclass